

ANR COLOSS conference "Arithmetic and geometry of locally symmetric spaces"

March 31-April 4th, UMPA, École Normale Supérieure de Lyon, Room 435 (and 411)

Organizers: Gabriel Dospinescu and Wiesława Nizioł

March 31 (Mon)

9:30-10:30 Christophe Breuil (CNRS, Paris-Saclay Univ.) : Hodge filtration and crystalline representations of $GL_n(\mathbf{Q}_p)$

11:00-12:00 Elmar Grosse-Klönne (Humboldt Univ., Berlin): Degeneration of families of modular local Galois representations

15:00-16:00 Cong Xue (CNRS, IMJ, Paris): Nearby cycles commute with proper direct image on stacks of shtukas

16:30-17:30 Hiroki Kato (IHES, Paris): Bernstein-Sato theory in positive characteristic and unit root nearby cycles

April 1st (Tue)

9:30-10:30 Juan Esteban Rodriguez-Camargo (Max Planck Institute for Mathematics, Bonn): Derived adic spaces and de Rham stacks

11:00-12:00 Guido Bosco (Max Planck Institute for Mathematics, Bonn): On the p -adic monodromy theorem

15:00-16:00 Zhixiang Wu (Münster Univ.): Bernstein-Zelevinsky duality for locally analytic principal series representations

16:30-17:30 Benjamin Schraen (Univ. Lyon 1): Exact functors on the category \mathcal{O} and patching

April 2nd (Wed)

9:30-10:30 Damien Junger (CNRS, Besançon Univ.): Integral line bundles of weight -1 on the Drinfeld half-plane and de Rham representations

11:00-12:00 Jean-François Dat (IMJ, Paris): Block decomposition and reduction to depth 0 for $\overline{\mathbf{Z}}[1/p]$ -representations of p -adic groups

April 3rd (Thu)

9:30-10:30 Olivier Taïbi (CNRS, ENS Lyon): Invariance of zeroes of Rankin-Selberg L-functions by Galois conjugation of the coefficients

11:00-12:00 Vincent Pilloni (CNRS, Paris-Saclay Univ.): p -adic Eichler-Shimura theory

15:00-16:00 Pierre Colmez (CNRS, IMJ Paris): Une approche localement analytique de la correspondance pour $\mathrm{GL}_2(\mathbf{Q}_p)$

April 4th (Fri)

9:30-10:30 Arnaud Eteve (Max Planck Institute for Mathematics, Bonn): Spectral action on isocrystals

11:00-12:00 Takeshi Saito (Tokyo Univ): Singular supports in positive and mixed characteristics

Abstracts:

Guido Bosco: On the p -adic monodromy theorem

I will present a new geometric perspective on the p -adic monodromy theorem of André, Kedlaya, and Mebkhout, which is based on the study of vector bundles on the analytic de Rham stack of the Fargues-Fontaine curve. I will then outline some applications to the p -adic Hodge theory of rigid-analytic varieties. This is based on joint work in progress with Anschütz, Le Bras, and Rodriguez Camargo.

Christophe Breuil: Hodge filtration and crystalline representations of $\mathrm{GL}_n(\mathbf{Q}_p)$

Let p be a prime number and $n \geq 2$ an integer. Let ρ be an n -dimensional crystalline representation of $\mathrm{Gal}(\overline{\mathbf{Q}_p}/\mathbf{Q}_p)$ with distinct Hodge-Tate weights and let $D := D_{\mathrm{cris}}(\rho)$ be its associated filtered φ -module. Under a very mild assumption on the eigenvalues of φ but crucially assuming that all the refinements on D are non-critical (equivalently that the Hodge filtration on D is as generic as possible), Y. Ding recently showed that, in a global context, the completed H^0 contains a finite length locally analytic representation $\pi(D)$ of $\mathrm{GL}_n(\mathbf{Q}_p)$ which determines and only depends on D (or equivalently ρ). But $\pi(D)$ remains somewhat mysterious. In this talk I will give an improved construction of the representation $\pi(D)$ which has 3 advantages: it has a more explicit relation with the Hodge filtration, it allows a much better understanding of the internal structure of $\pi(D)$ and it makes sense without assuming non-criticality, in particular gives a candidate for $\pi(D)$ in all cases. This is work in progress with Y. Ding.

Pierre Colmez: Une approche localement analytique de la correspondance pour $\mathrm{GL}_2(\mathbf{Q}_p)$

J'expliquerai comment construire la correspondance de Langlands locale p -adique pour $\mathrm{GL}_2(\mathbf{Q}_p)$ en commençant par la correspondance localement analytique au lieu de la caractéristique p (travail en commun avec Joaquin Rodrigues Jacinto).

Jean-François Dat: Block decomposition and reduction to depth 0 for $\overline{\mathbf{Z}}[1/p]$ -representations of p -adic groups

Scholze's motivic version of the Fargues-Scholze machinery produces a map from the ring of functions R on the stack of Langlands parameters over $\overline{\mathbf{Z}}[1/p]$ to the Bernstein center Z of the group with the same coefficients. In particular any idempotent in R produces a direct factor category of $\mathrm{Rep}(G)$. The primitive idempotents of R have been classified by DHKM and Sean Cotner. Assuming that p is "large", we will explain an explicit construction of (what should be) the associated direct factor category, using constructions of Yu and Kaletha. In general, this category will not be a block and we will decompose it further using additional cohomological data. Then we will report on a work in progress with J. Fintzen to prove that each factor so obtained is equivalent to the depth 0 category of another p -adic group canonically attached to the original primitive idempotent of R and the additional data. In particular, by a result in collaboration with Lanard, this implies that the factors we have constructed are blocks.

Arnaud Eteve: Spectral action on isocrystals

This is joint work in progress with Dennis Gaitsgory, Alain Genestier and Vincent Lafforgue. Let G be a reductive group over a local function field F . In their seminal work, Fargues and Scholze proposed a geometrization of the local Langlands correspondance for the pair (G, F) by constructing a "spectral action" on the category of ℓ -adic sheaves on Bun_G , the stack of G -torsors on the Fargues-Fontaine curve. The goal of this talk is to explain the construction of a different spectral action on the category of sheaves on the stack of G -isocrystals which should offer another geometrization of the local Langlands correspondance. Our construction has the benefit of being naturally compatible with the announced work of Hemo and Zhu and should also be equipped with a strong form of local-global compatibility.

Damien Junger: Integral line bundles of weight -1 on the Drinfeld half-plane and de Rham representations

The study of the ℓ -adic cohomology of the Drinfeld symmetric space and its coverings was central to exhibit geometric realisations of the classical local Langlands and Jacquet-Langlands correspondences. On the other hand, inspired by the paper of Dospinescu-Le Bras, the study of the coherent cohomology of equivariant vector bundles should allow us to realize some aspects of the p -adic local Langlands correspondance. More precisely, we will explain a work in progress which exhibits a class of equivariant line bundles "of weight -1 " that should provide a geometric realisation of some de dham Galois representations of Hodge-Tate weights $(0, 0)$.

Elmar Grosse-Klönne: Degeneration of families of modular local Galois representations

Let F/\mathbf{Q}_p and k/\mathbf{F}_p be finite extensions, let $n \in \mathbf{N}$. Using (φ, Γ) -modules, one can construct explicit families, parametrized by (block) triangular matrices in $\mathrm{GL}_n(k)$ (if

$F = \mathbb{Q}_p$), of (generic) Galois representations $\text{Gal}_F \rightarrow \text{GL}_n(k)$, such that the semisimplification of the restriction to inertia is constant. (If the family moves "in the lowest p -alcove", then, alternatively, one can use Fontaine-Laffaille theory instead.) One may attempt to extend these families further, yielding ("in the limit") Galois representations with *new* semisimplified restrictions to inertia. This directly relates to the intersection behaviour of the components of the Emerton-Gee stack.

Hiroki Kato: Bernstein-Sato theory in positive characteristic and unit root nearby cycles

I will talk about how to formulate (and an idea of a proof of) a positive characteristic analogue of the theorem of Kashiwara and Malgrange about the relationship, in characteristic zero, between the Bernstein-Sato polynomial and the monodromy eigenvalues of the nearby cycle sheaf. A key input is recent developments in the integral/mod p coefficient p -adic cohomology theory. This is a joint work in progress with Eamon Quinlan-Gallego and Daichi Takeuchi.

Vincent Pilloni: p -adic Eichler-Shimura theory

Abstract : Faltings proved the Hodge-Tate decomposition for modular curves, relating the p -adic étale cohomology and coherent cohomology. This result has been generalized to all Shimura varieties. In the p -adic Eichler-Shimura theory, one relates completed cohomology to overconvergent modular forms (or more generally higher Coleman theory). This talk is based on joint work with Boxer, Calegari, Gee (and earlier work of Pan and Rodriguez).

Juan Esteban Rodriguez-Camargo: Derived adic spaces and de Rham stacks

In this talk I will discuss a framework in Analytic Geometry generalizing Huber's theory of (analytic) adic spaces. Within this framework the analytic de Rham stack can be defined and satisfies very strong descent properties (such as proper proétale descent and arc-descent) making its construction only dependent on perfectoid spaces. This allows the definition of analytic de Rham stacks for (an arc variant of) diamonds, in particular for relative Fargues-Fontaine curves, leading to a stacky approach of the theory of Fargues-Fontaine de Rham cohomology previously constructed by Le Bras-Vezzani using motivic methods. This is based on previous work of the speaker and in work in progress with J. Anschütz, A-C Le Bras and P. Scholze, and J. Anschütz, G. Bosco and A-C Le Bras.

Takeshi Saito: Singular supports in positive and mixed characteristics

The singular support of a constructible sheaf on a smooth scheme over a field of positive characteristic was defined and proved to exist by Beilinson. After briefly recalling his theory, we discuss its analogue in mixed characteristics and the existence of a saturated relative variant.

Benjamin Schraen: Exact functors on the category \mathbf{O} and patching

Using finite slope locally analytic representations of a p -adic GL_n and p -adic automorphic forms on unitary groups, it is possible to construct a patching functor from the BGG category of $U(\mathfrak{gl}_n)$ -modules to a category of modules over a Galois deformation ring. When n is smaller than or equal to 3, we compare this functor to another functor

constructed by Bezrukavnikov from the BGG category to a category of coherent sheaves over the Steinberg variety. This is a joint work with Eugen Hellmann and Valentin Hernandez.

Olivier Taïbi: Invariance of zeroes of Rankin-Selberg L-functions by Galois conjugation of the coefficients

It follows from a conjecture of Deligne on special values of L -functions that vanishing of $L(1/2, \pi)$, for a cuspidal automorphic representation π for $GL_{N,F}$ (F a number field) satisfying an algebraicity condition at archimedean places, should be invariant under Galois conjugation of the coefficients. I will explain the proof of this invariance for certain Rankin-Selberg L-functions (under algebraicity and regularity assumptions). This is joint work with Laurent Clozel and Arno Kret that generalizes their previous result.

Zhixiang Wu: Bernstein-Zelevinsky duality for locally analytic principal series representations

I will talk about a calculation of the Bernstein-Zelevinsky dual of locally analytic principal series representations of p -adic Lie groups. The main ingredient of the calculation is Kohlhaase-Schraen's resolutions of the principal series representations by compactly induced representations. The resolutions also enable us to establish the duality of the coherent sheaves on the (patched) eigenvariety attached to these locally analytic representations. This is joint work with Matthias Strauch.

Cong Xue: Nearby cycles commute with proper direct image on stacks of shtukas

The proper direct image on stacks of shtukas for sheaves which indicate the level generalize the space of automorphic forms with level. In this talk, I will explain in the easiest case (stack of shtukas with one paw), why the nearby cycles commute with the proper direct image (even though the stacks of shtukas are not proper in general), for sheaves which indicate the level. I will also talk about the general statement and the relation with the local-global compatibility. This is a joint work with Arnaud Eteve.